Objective

Students should demonstrate a satisfactory understanding of fluid mechanics topics outlined below. A satisfactory understanding includes the ability to apply and utilize fundamental principles and concepts to solve fluid mechanics problems using appropriate assumptions and connections. Most undergraduate texts contain problems ranging from simple questions intended to reinforce the understanding of basic principles to more advanced problems requiring a synthesis of the principles and concepts and their applications to various engineering applications. Students should prepare for the qualifier by mastering these more advanced problems.

Instructions

Exam Format:

- This exam is comprised of 3 problems that can span the primary fluid mechanics topics covered in a typical undergraduate introductory fluid mechanics course in an ABET-accredited mechanical engineering program (ME EN 312 at BYU). Further details on topics are given below.
- This exam is open book and open notes with the exception of solution manuals of any kind, which are not allowed. Calculators may be used. Internet-connected or other communication devices are not permitted in the exam room. However, if access to an e-text is required, all communication methods must be disabled (e.g., wifi, Bluetooth, etc.)
- All work should be in neat engineering style with assumptions clearly stated. Partial credit may be awarded, but only in cases where the assumptions and solution approach are clearly explained in response to the questions. Start each problem solution on a separate sheet of paper.
- The student’s name should appear on each sheet of paper to be graded.
- A solution to each question provided should be completed.
- The exam has a time limit of 2 hours.
- A score of 70% or higher is considered a passing grade.
Exam Topics and Learning Objectives

Exam Topics:
For general exam preparation, review the ME Department *PhD Qualifying Exam General Information* document. The topics for the fluid mechanics qualifying exam include (see specific topics and descriptions below):

1. Fluid transport and physical properties
2. Fluid statics
3. Inviscid flows and acceleration
4. Conservation of mass
5. The linear momentum principle
6. Dimensional analysis
7. Internal flow
8. External flow

Study of these fluid mechanics topics in preparation for the exam may include completing and reviewing material from an undergraduate fluid mechanics course (e.g., ME EN 312 at BYU or equivalent) and reviewing fluid mechanics principles as found in the following reference or similar undergraduate texts: Munson, B. R., Okiishi, T. H., Huebsch, W. W., Rothmayer, A. P., 2013, *Fundamentals of Fluid Mechanics*, 7th ed., John Wiley and Sons, Inc.

The following outline of topics provides additional clarity on the scope of exam topics. Concepts provided here are NOT intended to be exhaustive, but instead, are included to represent topics in the discipline of fluid mechanics that the qualifying exam may include.

1. **Fluid Transport and Physical Properties** – Compute shear stress for flows involving Newtonian fluids, estimate fluid compressibility, determine forces due to surface tension, and estimate cavitation.

2. **Fluid Statics** – Determine pressure at a point in a static gas or liquid and compute hydrostatic forces and moments on fully and partially submerged surfaces.

3. **Inviscid Flows and Acceleration** - Evaluate local, convective and total acceleration, recognize Lagrangian and Eularian frames of reference, and appropriately apply Bernoulli’s principle.
4. **Conservation of Mass** - Derive the integral form of the conservation of mass principle from the Reynolds Transport Theorem and apply it to steady and unsteady flow situations in uniform or two-dimensional velocity distributions. Utilize the differential form of conservation of mass.

5. **Linear Momentum Principle** - Derive the integral form of the linear momentum principle from the Reynolds Transport Theorem and apply global force/momentum balances for stationary and constant velocity control volumes. Solve the Navier-Stokes equations for simplified viscous flows.

6. **Dimensional Analysis** - Determine appropriate dimensionless variables for a given dynamical situation and predict prototype dynamics based on similitude.

7. **Internal flow** - Apply the mechanical energy equation to laminar and turbulent flows with major and minor losses through pipe networks.

8. **External flow** - Predict velocity distributions, flat plate boundary layer flow, skin friction, and pressure drag for laminar and turbulent flows. Describe methods of drag reduction and lift dynamics.
SAMPLE PROBLEMS

The following pages provide sample problems that are representative of the type anticipated for the qualifying exam.

The five blades of a fan can be estimated as flat plates rotating at 90 rpm. Each blade is 0.5 m long and 0.1 m wide (as shown).

A. Estimate the torque needed to overcome the friction on the blades.
B. Estimate the torque needed to overcome the total drag.
C. Discuss the effects of your assumptions on the accuracy of your results. State all assumptions and show all your work.

A spherical object falls vertically from a great height. When the object reaches a terminal velocity of $V_s = 200 \text{ ft/sec}$, the air density is $\rho = 2.51 \times 10^{-3} \text{ slug/ft}^3$. The velocity profile of the wake behind the object is measured, with the results shown below. Atmospheric pressure acts uniformly in the region where the measurement is made. Find the drag force on the object. Show your work.
The flow over a Quonset hut may be approximated by the velocity field $\vec{V}$ given below, with $0 < \theta < \pi$:

$$\vec{V} = U \left[ 1 - \left( \frac{a}{r} \right)^2 \right] \cos \theta \hat{r} - U \left[ 1 + \left( \frac{a}{r} \right)^2 \right] \sin \theta \hat{\theta},$$

Where $r =$ radial coordinate away from the center of a cylinder of radius $a$, and $\hat{r}$ and $\hat{\theta}$ denote unit vectors in the $r$ and $\theta$ directions, respectively.

During a storm the wind speed reaches 90 km/hr and the outside temperature is 5° C. A barometer inside the hut reads 720 mm of mercury (mmHg). The pressure $\rho_{\infty}$ is also 720 mmHg. The hut has a diameter of 6 m and a length (out of the page) of 18 m. Assuming steady, incompressible, frictionless flow, determine the net force tending to lift the hut off its foundation. Show your work.
A jet impinges on a cart with velocity \( V_j = 1 \text{ m/s} \) and area \( A_j = 0.1 \text{ m}^2 \). The cart can hold water, but has a spout which lets water exit at 60° from the horizontal. Given the values shown in the figure below and assuming the water level in the cart stays constant at \( H \), find:

A. The force necessary to hold the cart in place and whether you think this is an over- or under-estimate of the actual force necessary.
B. The force necessary to allow the cart to move to the right at 0.25 m/s.
C. Now assume the cart starts empty with \( H \) increasing, set up (Do Not Solve Them) the differential equations necessary to find the acceleration of the cart during the transient state if no restraining force is applied.

The radial variation of velocity at the midsection of a 180° bend is given by \( rV_\theta = \text{constant} \). The cross section of the bend is square (i.e. the depth of the channel into the paper is \( R_2 - R_1 \)). Derive an equation for the pressure difference between the outside and the inside of the bend \( (p_2 - p_1) \). Express your answer in terms of the mass flow rate \( \dot{m} \), the fluid density \( \rho \), the geometric parameters \( R_1 \) and \( R_2 \) and the depth of the bend \( h = R_2 - R_1 \).

You may assume the following: the flow is frictionless, velocity is not a function of \( z \) (the coordinate into the paper), density is constant, and streamlines are circular in the bend.
Consider a double layer of immiscible fluids 1 and 2, flowing steadily down an inclined plane, as shown below. The atmosphere exerts no shear stress on the surface and is at constant pressure. There is no flow in the z-direction. Find the velocity profiles in fluids 1 and 2 ($u_1$ and $u_2$). Express your answer in terms of gravity ($g$), angle ($\theta$), densities ($\rho_1, \rho_2$), viscosities ($\mu_1, \mu_2$), and film thickness ($h_1, h_2$).

An air bubble of initial diameter 1 mm is formed at a depth of 30 m below the surface of a lake (25°C). The bubble rises isothermally and expands as it rises due to the decreasing pressure. You may neglect any pressure difference between the water and the air inside of the bubble due to surface tension. Also, assume that the bubble remains spherical in shape as it rises and that the drag of the bubble is the same as that for a solid sphere, with the drag coefficient staying constant at the initial value. Lastly, the weight and inertia of the bubble are negligible compared to the other forces that act on it. Estimate the initial speed of the bubble, the diameter of the bubble at a depth of 5 m, the speed of the bubble at this depth, and the time required for the bubble to reach a depth of 5 m.

Air flows in a cylindrical duct of diameter $D=150$ mm. At section 1, the turbulent boundary layer is of thickness $\delta_1 = 10$ mm and the velocity in the inviscid central core is $u_1 = 25$ m/s. Further downstream, at section 2, the boundary layer is of thickness $\delta_2 = 30$ mm. You may assume that the boundary layer velocity profile is approximated well by the 1/7-power expression.

A. Calculate the velocity, $u_2$, in the inviscid central core at the second section.
B. Estimate the length of duct between sections 1 and 2. For this you may assume that there is a zero pressure gradient.
C. Estimate the distance at which the boundary layer thickness is 20 mm. For this you may also assume that there is a zero pressure gradient.

The siphon shown is fabricated from 2-inch (inside diameter) drawn aluminum tubing and is flowing. The liquid is water at 60°F.

A. Compute the volume flow rate through the siphon.
B. Estimate the minimum pressure inside of the tube.

Water is circulated from a large tank, through a filter, and back to the tank as shown below. The power added to the water by the pump is 200 ft.lbf/s. All pipe diameters are 0.1 ft and made of concrete (the smoothest type). The valve is a swing-check valve and all elbows are threaded. Other losses are as shown on the figure. **Determine the flowrate through the filter.**

An unknown liquid of height “h” is flowing down a slope with angle \( \theta \) from the horizontal. What non-dimensional parameters are important for describing this flow (show all work)?

*Note: you should have 2 recognizable common non-dimensional parameters. If the boundary
were sinusoidal what new non-dimensional groups would be introduced? Assuming it is possible to create a half scale model of this flow, what is the relationship between the drag force in the model and the full scale flow? If the full scale fluid is glycerin, does a half scale model seem plausible?